THE SECOND MAIN THEOREM OF HOLOMORPHIC CURVES INTO PROJECTIVE SPACES

PEI-CHU HU AND CHUNG-CHUN YANG

ABSTRACT. By utilizing Jacobian sections introduced by Stoll, we prove a second main theorem of holomorphic curves into projective spaces for hypersurfaces under certain conditions on the jets of the curves. The precise conditions are listed in three equations in the introduction. This solves a weaker form of Griffiths' conjecture and only for a special class of holomorphic curves.

1. INTRODUCTION

We will study a non-constant holomorphic curve

$$f = [f_0, \dots, f_n] : \mathbb{C} \longrightarrow \mathbb{P}^n = \mathbb{P}^n(\mathbb{C}),$$

where $f_0, \dots, f_n$ are entire functions on $\mathbb{C}$ without common zeros. Without loss of generality, we may assume $f_0 \not\equiv 0$. Define

$$g_j = \frac{f_j}{f_0}, \quad j = 1, \dots, n.$$

For any positive integer k , we obtain a holomorphic curve

$$(1) \quad f_k = [1, g_1^{(k)}, \dots, g_n^{(k)}] : \mathbb{C} \longrightarrow \mathbb{P}^n.$$

Let z be the coordinate of $\mathbb{C}$ and let f' be the derivative of f with respect to z which is a mapping from $\mathbb{C}$ to the holomorphic tangent bundle $\mathbf{T}(\mathbb{P}^n)$ of $\mathbb{P}^n$. Let H be the hyperplane section line bundle over the projective space $\mathbb{P}^n$. For a positive integer d , let H^d be the d -th power of H . Let $\Gamma(\mathbb{P}^n, H^d)$ be the vector space of holomorphic sections of H^d over $\mathbb{P}^n$. Now we state our main theorem:

Theorem 1.1. *Take*

$$s_j \in \Gamma(\mathbb{P}^n, H^d) - \{0\}, \quad j = 1, \dots, q,$$

such that the divisor $D = D_1 + \dots + D_q$ has simple normal crossings in $\mathbb{P}^n$, where $D_j = (s_j = 0)$ is the zero divisor of s_j . Let $f : \mathbb{C} \longrightarrow \mathbb{P}^n$ be a non-constant holomorphic curve such that

$$f(\mathbb{C}) \not\subseteq \text{supp}(D_j), \quad f_k(\mathbb{C}) \not\subseteq \text{supp}(D_j), \quad j = 1, \dots, q, \quad k = 1, \dots, n-1.$$

Received by the editors September 16, 2007 and, in revised form, December 17, 2009.

2010 *Mathematics Subject Classification.* Primary 32H02, 32H25.

Key words and phrases. Holomorphic curve, projective space, line bundle, Jacobian section, value distribution theory.

The work of the first author was partially supported by the Natural Science Foundation of China and Shandong.

The second author was partially supported by a UGC Grant of Hong Kong: Project No. 604103.

Assume further that there exist homomorphisms

$$(2) \quad t_k : f_k^*(\mathbf{T}(\mathbb{P}^n)) \longrightarrow f^*(\mathbf{T}(\mathbb{P}^n)), \quad k = 1, \dots, n-1,$$

such that lifted sections f'_{k,f_k} of f'_k for f_k satisfy

$$(3) \quad f'_f \wedge t_1(f'_{1,f_1}) \wedge \cdots \wedge t_{n-1}(f'_{n-1,f_{n-1}}) \neq 0.$$

Then for any $\varepsilon > 0$,

$$(4) \quad \begin{aligned} \| \quad (qd - n - 1)T_f(r) &\leq \sum_{j=1}^q N_f(r, D_j) - N_{\text{Ram}}(r, f) \\ &+ \sum_{k=1}^{n-1} Y_{t_k}(r) + O(\log^+ T_f(r)) + O(\varepsilon \log r), \end{aligned}$$

where the symbol $\|$ at the front of an inequality denotes that the inequality holds as $r \rightarrow \infty$ outside of a possible exceptional set E with $\int_E dr/r < \infty$.

For the notation and symbols in Theorem 1.1, see Section 2 and (20). If we define the defect of f for D_j ,

$$\delta_f(D_j) = 1 - \limsup_{r \rightarrow \infty} \frac{N_f(r, D_j)}{dT_f(r)},$$

and if the homomorphisms (2) satisfy *admissible* conditions

$$(5) \quad \|_{\varepsilon} \quad Y_{t_k}(r) \leq \varepsilon T_f(r), \quad k = 1, \dots, n-1,$$

for $\varepsilon > 0$, then (4) immediately yields the defect relation

$$(6) \quad \sum_{j=1}^q \delta_f(D_j) \leq \frac{n+1}{d}.$$

P. Griffiths ([13], or cf. [23]) conjectured that (4) or (6) holds if f is algebraically non-degenerate. By the definition, a mapping $f : \mathbb{C} \rightarrow \mathbb{P}^n$ is algebraically non-degenerate if the image of f is not contained in any proper subvariety of $\mathbb{P}^n$. A. Biancofiore [1] proved (6) for a class of holomorphic curves. For holomorphic curves $f : \mathbb{C} \rightarrow \mathbb{P}^2$, Siu [24] proved (6) with $n+1=3$ by using meromorphic connections. Theorem 1.1 mainly gives a special class of holomorphic curves satisfying (2), (3) and (5) such that (6) holds.

Special cases related to Theorem 1.1 and (6), namely, $d=1$, were studied by Bloch [2], Cartan [4] and Dufresnoy [6], or see Fujimoto [7], [8], Green [9], [10], [11], Kobayashi [20], Lang [21], Wong-Wong [35], Wu [38], and Wang [34] for an analogue over function fields.

These results may be regarded as special cases of the following general problem:

Conjecture 1.2. *Let N be an algebraic manifold of dimension n and let D be a divisor on N with normal crossings. There exists a proper algebraic subset Z_D of N having the following property. Let $f : \mathbb{C} \rightarrow N$ be a holomorphic curve such that $f(\mathbb{C}) \not\subset Z_D$. Let E be an ample divisor. Then*

$$(7) \quad \| \quad T_f(r, [D]) + T_f(r, K_N) \leq N_f(r, D) + O\{\log(rT_f(r, [E]))\}.$$

In this form the conjecture is due to S. Lang (cf. [21]). It includes the conjecture of P. Griffiths ([13] and [23]) as a special case. When N is an Abelian variety, Siu and Yeung (cf. [27], [28]) solved Conjecture 1.2 in the affirmative. Generally, Siu (cf. [24], [25], [26], or [22]) gave some approaches by using meromorphic connections. See Vojta [32] for some new progress.

We note that the Griffiths and Lang conjectures have been translated into a general conjecture in Diophantine approximation by P. Vojta [31] and into a conjecture for holomorphic curves over non-Archimedean fields by Hu and Yang [18]. Theorem 1.1 provides new supportive evidence for these conjectures.

2. PRELIMINARIES

We briefly recall the concept of a Jacobian bundle of Jacobian sections first introduced and studied by W. Stoll (see [29] and [30]). Let N be a smooth projective algebraic variety of dimension n . Take a positive integer m with $n > m \geq 1$. Let M be a complex manifold of dimension m and consider a holomorphic mapping

$$f : M \longrightarrow N.$$

Let $\pi : E \longrightarrow N$ be a holomorphic vector bundle over N . The vector space of holomorphic sections of E over N is denoted by $\Gamma(N, E)$. The *pullback bundle* $\tilde{\pi} : f^*(E) \longrightarrow M$ is defined only up to an isomorphism such that $f \circ \tilde{\pi} = \pi \circ \tilde{f}$, where $\tilde{\pi}$ is the bundle projection and $\tilde{f} : f^*(E) \longrightarrow E$ is a bundle homomorphism over f . If $s \in \Gamma(N, E)$, then a *lifted section* $s_f \in \Gamma(M, f^*(E))$ of s for f is uniquely defined by

$$s_f(x) = \tilde{f}_x^{-1}(s(f(x))), \quad x \in M.$$

Assume $s \neq 0$ and $f(M) \not\subset s^{-1}(0)$. The lifted section s_f exists with $s_f \neq 0$.

Let $\mathbf{T}^*(N)$ be the dual of $\mathbf{T}(N)$. The *canonical bundle* of N is defined by

$$K_N = \bigwedge_n \mathbf{T}^*(N).$$

According to Stoll [29], the bundle

$$K(f) = K_M \otimes f^*(K_N^*)$$

is called the *Jacobian bundle*, where K_N^* is the dual of K_N . A holomorphic section F of $K(f)$ over M is said to be a *Jacobian section*. The section F is called *effective* if $F^{-1}(0)$ is thin. The zero divisor (F) of F is called the *ramification divisor* of f for F .

Let F be a Jacobian section and U be an open subset in N such that $\tilde{U} = f^{-1}(U) \neq \emptyset$. The inner product

$$\langle \cdot, \cdot \rangle : K_N^* \oplus K_N \longrightarrow \mathbb{C}$$

is well defined and pulls back to

$$\langle \cdot, \cdot \rangle : f^*(K_N^*) \oplus f^*(K_N) \longrightarrow \mathbb{C},$$

which further induces a K_M -valued inner product

$$\langle \cdot, \cdot \rangle : K(f) \oplus f^*(K_N) \longrightarrow K_M.$$

The section F defines a linear mapping

$$F : \Gamma(U, K_N) \longrightarrow \Gamma(\tilde{U}, K_M)$$

by $F[\psi] = \langle F, \psi_f \rangle$ for all $\psi \in \Gamma(U, K_N)$ and where ψ_f is the lifted section defined previously. If $(V; z_1, \dots, z_m)$ and $(W; w_1, \dots, w_n)$ are holomorphic coordinate charts on M and N respectively with $W \subset U$ and $f(V) \subset W$, then $\psi = \psi_W dw$ on W and $F = F_{VW} dz \otimes d^* w_f$ on V , where

$$dw = dw_1 \wedge \cdots \wedge dw_n, \quad d^* w = \frac{\partial}{\partial w_1} \wedge \cdots \wedge \frac{\partial}{\partial w_n},$$

and where ψ_W and F_{VW} are holomorphic functions. Then on V we have

$$F[\psi] = F_{VW}(\psi_W \circ f) dz.$$

Set

$$i_p = \left(\frac{\sqrt{-1}}{2\pi} \right)^p (-1)^{\frac{p(p+1)}{2}} p!.$$

Given a Jacobian section $F \in \Gamma(M, K(f))$, then a linear mapping

$$F : A^{2n}(U) \longrightarrow A^{2m}(\tilde{U})$$

is uniquely defined as follows: If $\Psi \in A^{2n}(U)$ is expressed by

$$\Psi = i_n \Psi_W dw \wedge \overline{dw}$$

in terms of the holomorphic coordinate chart $(W; w_1, \dots, w_n)$ on N , then

$$F[\Psi]|_{\tilde{W}} = i_m(\Psi_W \circ f) F[dw] \wedge \overline{F[dw]}.$$

Note that a homomorphism

$$\hat{f} : f^* \left(\bigwedge_m \mathbf{T}^*(N) \right) \longrightarrow \bigwedge_m \mathbf{T}^*(M)$$

exists uniquely such that $\hat{f}(\psi_f) = f^*(\psi)$ for all $\psi \in \Gamma \left(U, \bigwedge_m \mathbf{T}^*(N) \right)$, where U is open in N with $\tilde{U} = f^{-1}(U) \neq \emptyset$. Set $q = n - m$. The interior product

$$\angle : K_N \oplus \bigwedge_q \mathbf{T}(N) \longrightarrow \bigwedge_m \mathbf{T}^*(N)$$

also pulls back to an interior product

$$\angle : f^*(K_N) \oplus f^* \left(\bigwedge_q \mathbf{T}(N) \right) \longrightarrow f^* \left(\bigwedge_m \mathbf{T}^*(N) \right).$$

Take a holomorphic section $\varphi \in \Gamma \left(M, f^* \left(\bigwedge_q \mathbf{T}(N) \right) \right)$, which is called a *holomorphic field on f* over M of degree q . Then φ induces a Jacobian section F_φ defined by

$$F_\varphi|_{\tilde{U}} = \hat{f}(\chi_f \angle \varphi) \otimes \chi_f^*,$$

where $\chi \in \Gamma(U, K_N)$ vanishes nowhere for the open subset of N with $\tilde{U} = f^{-1}(U) \neq \emptyset$ and χ^* is the dual frame. Equivalently, F_φ can be described by its action

$$F_\varphi[\psi] = \hat{f}(\psi_f \angle \varphi)$$

for all $\psi \in \Gamma(U, K_N)$, where U is open in N with $\tilde{U} = f^{-1}(U) \neq \emptyset$. The section φ is said to be *effective* for f if F_φ is effective.

Next we assume $M = \mathbb{C}^m$ with the standard holomorphic coordinates $z = (z_1, \dots, z_m)$. Define the exhaustion function $\tau : \mathbb{C}^m \longrightarrow [0, +\infty)$ by

$$\tau(z) = \|z\|^2 = \sum_{j=1}^m |z_j|^2$$

and set

$$v = dd^c \tau, \quad \sigma = d^c \log \tau \wedge (dd^c \log \tau)^{m-1},$$

where

$$d^c = \frac{\sqrt{-1}}{4\pi}(\bar{\partial} - \partial).$$

Take $r > 0$. Define

$$\mathbb{C}^m \langle 0; r \rangle = \{z \in \mathbb{C}^m \mid \tau(z) = r^2\}, \quad \mathbb{C}^m [0; r] = \{z \in \mathbb{C}^m \mid \tau(z) \leq r^2\}.$$

If h is an integrable function over $\mathbb{C}^m \langle 0; r \rangle$, we write

$$\mathbb{C}^m \langle 0; r; h \rangle = \int_{\mathbb{C}^m \langle 0; r \rangle} h \sigma.$$

For an integrable $(1, 1)$ -form φ on $\mathbb{C}^m$, set

$$\mathbb{C}^m [0; r; \varphi] = r^{2-2m} \int_{\mathbb{C}^m [0; r]} \varphi \wedge v^{m-1}$$

and define

$$\mathcal{T}(r, r_0; \varphi) = \int_{r_0}^r \frac{\mathbb{C}^m [0; t; \varphi]}{t} dt$$

for a fixed $r_0 > 0$ with $r > r_0$.

Let ν be a multiplicity on $\mathbb{C}^m$. For $t > 0$, the *counting function* n_ν is defined by

$$(8) \quad n_\nu(t) = t^{2-2m} \int_{\mathbb{C}^m [0; t] \cap A} \nu v^{m-1},$$

where $A = \text{supp } \nu$. Here, if $m = 1$, we define

$$(9) \quad n_\nu(t) = \sum_{z \in \mathbb{C} [0; t] \cap A} \nu(z).$$

If ν is non-negative, then n_ν increases. The *valence function* of ν is defined by

$$(10) \quad N_\nu(r) = N_\nu(r, r_0) = \int_{r_0}^r \frac{n_\nu(t)}{t} dt \quad (r \geq r_0).$$

Lemma 2.1 ([16], [19]). *Define*

$$Z_i = f_* \left(\frac{\partial}{\partial z_i} \right), \quad i = 1, \dots, m.$$

Then a holomorphic field φ on f over $\mathbb{C}^m$ of degree $n - m$ is such that φ is effective for f if and only if

$$Z = Z_{1f} \wedge \cdots \wedge Z_{mf} \wedge \varphi \neq 0.$$

Further, we have $(F_\varphi = 0) = (Z = 0)$.

Lemma 2.2 ([16], [19]). *Let φ be an effective holomorphic field on f over $\mathbb{C}^m$ of degree $n - m$. Define a non-negative function g by*

$$F_\varphi[\psi^n] = g^2 f^*(\psi^m),$$

where ψ is the associated 2-form of a Hermitian metric on N . Then $g \leq |\varphi|$, where $|\cdot|$ is the norm induced by the metric ψ .

Lemma 2.3 ([15], [19]). *Let $\eta \geq 0$ be a form of bidegree $(1, 1)$ on $\mathbb{C}^m$ such that $\mathcal{T}(r, r_0; \eta)$ exists for $r > r_0 > 0$. Let u be a non-negative function on $\mathbb{C}^m$ satisfying*

$$uv^m \leq \eta \wedge v^{m-1}.$$

Then for any $\varepsilon > 0$,

$$\| \mathbb{C}^m \langle 0; r; \log u \rangle \leq (1 + 2\varepsilon) \log \mathcal{T}(r, r_0; \eta) + 4\varepsilon \log r.$$

Let L be a line bundle over N . Take $0 \neq s_j \in \Gamma(N, L), j = 1, \dots, q$, such that the divisor $D = D_1 + \dots + D_q$ has normal crossings in N , where $D_j = (s_j = 0)$ is irreducible. Assume that each D_j ($1 \leq j \leq q$) is smooth, i.e., D has simple normal crossings, and a positive number λ exists such that Chern classes of L and K_N in de Rham cohomology $H_{\text{DR}}^2(N, \mathbb{R})$ satisfy

$$(11) \quad \lambda q c_1(L) + c_1(K_N) > 0.$$

Obviously, such λ exist if L is positive. Then there are a volume form Ω on N and a metric κ of L such that

$$(12) \quad 0 \leq |s_j|_\kappa^2 < \frac{1}{e}, \quad j = 1, \dots, q,$$

$$(13) \quad \lambda q c_1(L, \kappa) + \text{Ric}(\Omega) > 0$$

and such that the form

$$(14) \quad \Psi = \frac{\Omega}{\prod_{j=1}^q |s_j|_\kappa^{2\lambda} (\log |s_j|_\kappa^2)^2},$$

called a *Carlson-Griffiths form*, satisfies

$$(15) \quad \text{Ric}(\Psi) > 0, \quad \text{Ric}(\Psi)^n \geq \frac{\Omega}{\prod_{j=1}^q |s_j|_\kappa^{2\lambda} (\log |s_j|_\kappa^2)^2}, \quad \int_N \text{Ric}(\Psi)^n < \infty;$$

see Carlson-Griffiths [3], Griffiths-King [14] and Stoll [29].

Lemma 2.4 ([3], [14], [29]). *Let $L, D = D_1 + \dots + D_q, \lambda$ and Ψ be as defined above. Assume that condition (12) holds. Let $f : \mathbb{C}^m \rightarrow N$ be a holomorphic mapping satisfying the condition $f(\mathbb{C}^m) \not\subseteq \text{supp}(D_j)$ for each $j = 1, \dots, q$. Then there exist a constant $c_1 \leq 0$ such that for $r > r_0$,*

$$(16) \quad c_1 \leq \lambda q T_f(r, L) + T_f(r, K_N) - \mathcal{T}(r, r_0; f^*(\text{Ric}(\Psi))) \leq q \log T_f(r, L) + O(1),$$

where

$$T_f(r, L) = \mathcal{T}(r, r_0; f^*(c_1(L, \kappa))), \quad T_f(r, K_N) = \mathcal{T}(r, r_0; f^*(\text{Ric}(\Omega)))$$

are well defined, up to $O(1)$.

Theorem 2.5 ([16]). *Assume that N is a compact complex manifold of dimension n . Let L be a positive holomorphic line bundle on N . Take $0 \neq s_j \in \Gamma(N, L), j = 1, \dots, q$, such that the divisor $D = D_1 + \dots + D_q$ has simple normal crossings in N , where $D_j = (s_j = 0)$ is irreducible for each j . Assume that a positive number λ exists such that Chern classes of L and K_N in de Rham cohomology $H_{\text{DR}}^2(N, \mathbb{R})$ satisfy (11). Let $f : \mathbb{C}^m \rightarrow N$ be a holomorphic mapping such that $f(\mathbb{C}^m) \not\subseteq s_j^{-1}(0)$ for all j . Assume that there exists an effective Jacobian section F of f . Let g be the function defined by*

$$F[\text{Ric}(\Psi)^n] = g^2 f^*(\text{Ric}(\Psi)^m),$$

where Ψ is the Carlson-Griffiths form defined in (14). Then for any $\varepsilon > 0$,

$$(17) \quad \begin{aligned} \| \quad qT_f(r, L) + T_f(r, K_N) &\leq \sum_{j=1}^q N_f(r, D_j) - N_{\mu_{(F)}}(r) + \mathbb{C}^m \langle 0; r; \log g \rangle \\ &+ O(\log^+ T_f(r, L)) + O(\varepsilon \log r), \end{aligned}$$

where $\mu_{(F)}$ is the associated multiplicity of the zero divisor $(F = 0)$ of F over $\mathbb{C}^m$ (see [29] or [17]), and where

$$N_f(r, D_j) = N_{\mu_{f^*D_j}}(r), \quad \log^+ T_f(r, L) = \log \max\{1, T_f(r, L)\}.$$

For an effective Jacobian section F of f , we write

$$N_{\text{Ram}}(r, f) = N_{\mu_{(F)}}(r).$$

In terms of a local coordinate system $w_1, \dots, w_n$ of N we may write

$$\text{Ric}(\Psi) = \frac{\sqrt{-1}}{2\pi} \sum_{\alpha, \beta} h_{\alpha\beta} dw_\alpha \wedge d\bar{w}_\beta.$$

The singular form $\text{Ric}(\Psi)$ is smooth outside the divisor D , and hence defines a smooth metric

$$h_\Psi = 2 \sum_{\alpha, \beta} h_{\alpha\beta} dw_\alpha d\bar{w}_\beta$$

which has “singularity” on D .

Theorem 2.6 ([19]). *If F is an effective Jacobian section of f and if $f(\mathbb{C}^m) \not\subseteq B(L) \neq N$, then there exists a positive constant c such that*

$$\| \quad cT_f(r, L) < \mathbb{C}^m \langle 0; r; \log g \rangle + O(\varepsilon \log r)$$

holds for any $\varepsilon > 0$, where g is defined by

$$F[c_1(L, \kappa)^n] = g^2 f^*(c_1(L, \kappa)^m).$$

3. PROOF OF THEOREM 1.1

Now we show that Theorem 1.1 follows from the result:

Theorem 3.1. *Take*

$$s_j \in \Gamma(\mathbb{P}^n, H^d) - \{0\}, \quad j = 1, \dots, q,$$

such that the divisor $D = D_1 + \dots + D_q$ has simple normal crossings in $\mathbb{P}^n$, where $D_j = (s_j)$ is the zero divisor of s_j . Let $f : \mathbb{C} \rightarrow \mathbb{P}^n$ be a non-constant holomorphic curve such that

$$f(\mathbb{C}) \not\subseteq \text{supp}(D_j), \quad j = 1, \dots, q.$$

Assume that there exist holomorphic curves $f_k : \mathbb{C} \rightarrow \mathbb{P}^n$ with

$$f_k(\mathbb{C}) \not\subseteq \text{supp}(D_j), \quad j = 1, \dots, q, \quad k = 1, \dots, n-1,$$

and homomorphisms

$$t_k : f_k^*(\mathbf{T}(\mathbb{P}^n)) \rightarrow f^*(\mathbf{T}(\mathbb{P}^n)), \quad k = 1, \dots, n-1,$$

satisfying

$$f'_f \wedge t_1(f'_{1, f_1}) \wedge \dots \wedge t_{n-1}(f'_{n-1, f_{n-1}}) \neq 0.$$

Then for any $\varepsilon > 0$,

$$(18) \quad \begin{aligned} \| \quad (qd - n - 1)T_f(r) &\leq \sum_{j=1}^q N_f(r, D_j) - N_{\text{Ram}}(r, f) + \sum_{k=1}^{n-1} Y_{t_k}(r) \\ &+ \left(\frac{1}{2} + \varepsilon\right) \sum_{k=1}^{n-1} \log T_{f_k}(r) + O(\log^+ T_f(r)) + O(\varepsilon \log r). \end{aligned}$$

Proof. We will apply Theorem 2.5 to prove Theorem 3.1. To do this, we take

$$m = 1, \quad N = \mathbb{P}^n, \quad L = H^d,$$

and abbreviate

$$(19) \quad T_f(r) = T_f(r, H).$$

By assumption, the holomorphic field

$$\varphi = t_1(f'_{1,f_1}) \wedge \cdots \wedge t_{n-1}(f'_{n-1,f_{n-1}})$$

is effective for f . We get, via Lemma 2.1, an effective Jacobian section F_φ of f . According to Theorem 2.5, a non-negative function g is defined by

$$F_\varphi[\psi^n] = g^2 f^*(\psi),$$

where $\psi = \text{Ric}(\Psi)$. By Lemma 2.2, we have the estimate $g \leq |\varphi|$, where $|\varphi|$ is measured by the induced metric from h_Ψ .

It is easy to show that there exists a constant $c > 0$ such that (cf. [19], Lemma 1.55)

$$|\varphi| \leq c |t_1(f'_{1,f_1})| \cdots |t_{n-1}(f'_{n-1,f_{n-1}})|.$$

Setting

$$(20) \quad Y_{t_k}(r) = \mathbb{C} \left\langle 0; r; \log \frac{|t_k(f'_{k,f_k})|}{|f'_k|} \right\rangle,$$

we obtain

$$\mathbb{C} \langle 0; r; \log g \rangle \leq \mathbb{C} \langle 0; r; \log |\varphi| \rangle \leq \sum_{k=1}^{n-1} Y_{t_k}(r) + \frac{1}{2} \sum_{k=1}^{n-1} \mathbb{C} \langle 0; r; \log |f'_k|^2 \rangle + O(1).$$

Further, Lemma 2.3 implies

$$\| \quad \mathbb{C} \langle 0; r; \log |f'_k|^2 \rangle \leq (1 + 2\varepsilon) \log \mathcal{T}(r, r_0; |f'_k|^2 v) + 4\varepsilon \log r.$$

Note that

$$\mathcal{T}(r, r_0; |f'_k|^2 v) = \mathcal{T}(r, r_0; f_k^*(\psi)).$$

By Lemma 2.4, there exist constants $c_1 \leq 0$, $c_2 \geq 0$, $c_3 \geq 0$ such that for $r > r_0$,

$$(21) \quad \begin{aligned} c_1 &\leq \lambda q T_{f_k}(r, L) + T_{f_k}(r, K_N) - \mathcal{T}(r, r_0; f_k^*(\psi)) \\ &\leq q \log \{T_{f_k}(r, L) + c_2\} + c_3. \end{aligned}$$

Since $L = H^d$, $N = \mathbb{P}^n$, we have

$$T_{f_k}(r, L) = dT_{f_k}(r, H) = dT_{f_k}(r), \quad T_{f_k}(r, K_N) = -(n+1)T_{f_k}(r).$$

Then we obtain

$$\mathcal{T}(r, r_0; f_k^*(\psi)) \leq (\lambda q d - n - 1)T_{f_k}(r) - c_1.$$

Therefore

$$\| \mathbb{C}\langle 0; r; \log g \rangle \leq \sum_{k=1}^{n-1} Y_{t_k}(r) + \left(\frac{1}{2} + \varepsilon \right) \sum_{k=1}^{n-1} \log T_{f_k}(r) + 2(n-1)\varepsilon \log r + O(1),$$

and so (18) follows from Theorem 2.5. $\square$

To prove Theorem 1.1, we need to estimate the characteristic functions $T_{f_k}(r)$ of holomorphic curves (1). Note that $f_0^{k+1} \mathbf{g}_j^{(k)}$ are entire functions such that f_k can be expressed as follows:

$$f_k = \left[f_0^{k+1}, f_0^{k+1} \mathbf{g}_1^{(k)}, \dots, f_0^{k+1} \mathbf{g}_n^{(k)} \right].$$

Applying a formula due to Stoll (cf. [30], (6.63), or see [19], (2.3.20)),

$$T_{f_k}(r) = \frac{1}{2} \mathbb{C} \left\langle 0; r; \log |f_0^{k+1}|^2 \left(1 + \sum_{j=1}^n |\mathbf{g}_j^{(k)}|^2 \right) \right\rangle - N_{\nu_k}(r) + O(1),$$

where ν_k is the multiplicity of common zeros of $f_0^{k+1}, f_0^{k+1} \mathbf{g}_1^{(k)}, \dots, f_0^{k+1} \mathbf{g}_n^{(k)}$, we easily obtain the inequality

$$\begin{aligned} T_{f_k}(r) &\leq \frac{1}{2} \mathbb{C} \left\langle 0; r; \log |f_0^{k+1}|^2 \left(1 + |\mathbf{g}_1^{(k)}|^2 + \dots + |\mathbf{g}_n^{(k)}|^2 \right) \right\rangle + O(1) \\ &\leq (k+1)T_f(r) + \frac{1}{2} \mathbb{C} \left\langle 0; r; \log \left(1 + \left| \frac{\mathbf{g}_1^{(k)}}{\mathbf{g}_1} \right|^2 + \dots + \left| \frac{\mathbf{g}_n^{(k)}}{\mathbf{g}_n} \right|^2 \right) \right\rangle + O(1). \end{aligned}$$

Take $0 < \alpha < 1/(2k)$. We have

$$\begin{aligned} \frac{1}{2} \mathbb{C} \left\langle 0; r; \log \left(1 + \sum_j \left| \frac{\mathbf{g}_j^{(k)}}{\mathbf{g}_j} \right|^2 \right) \right\rangle &= \frac{1}{\alpha} \mathbb{C} \left\langle 0; r; \log \left(1 + \sum_j \left| \frac{\mathbf{g}_j^{(k)}}{\mathbf{g}_j} \right|^2 \right)^{\frac{\alpha}{2}} \right\rangle \\ &\leq \frac{1}{\alpha} \log \left\{ 1 + \sum_j \mathbb{C} \left\langle 0; r; \left| \frac{\mathbf{g}_j^{(k)}}{\mathbf{g}_j} \right|^\alpha \right\rangle \right\}. \end{aligned}$$

Note that for any α with $0 < \alpha < \frac{1}{2k}$, the lemma of logarithmic derivative (cf. [39] or [19], Lemma 2.47) implies that there is a constant $C > 1$ such that for any $r_0 < r < \rho < R$,

$$\mathbb{C} \left\langle 0; r; \left| \frac{\mathbf{g}_j^{(k)}}{\mathbf{g}_j} \right|^\alpha \right\rangle \leq C \left\{ \frac{\rho T(R, \mathbf{g}_j)}{r(\rho - r)} \right\}^{k\alpha}, \quad j = 1, \dots, n,$$

where $T(R, \mathbf{g}_j)$ is Nevanlinna's characteristic function of $\mathbf{g}_j$. It is well known that (cf. [19], Lemma 2.37)

$$T(R, \mathbf{g}_j) \leq T_f(R) + O(1), \quad j = 1, \dots, n.$$

Therefore

$$\frac{1}{2} \mathbb{C} \left\langle 0; r; \log \left(1 + \sum_j \left| \frac{\mathbf{g}_j^{(k)}}{\mathbf{g}_j} \right|^2 \right) \right\rangle \leq k \log \left\{ \frac{\rho T_f(R)}{r(\rho - r)} \right\} + O(1)$$

for any $r_0 < r < \rho < R$. Hence (cf. [19], (2.5.10), or [39])

$$\| \frac{1}{2} \mathbb{C} \left\langle 0; r; \log \left(1 + \sum_j \left| \frac{\mathfrak{g}_j^{(k)}}{\mathfrak{g}_j} \right|^2 \right) \right\rangle \leq k \log T_f(r) + k(1 + \varepsilon) \log \log T_f(r) + O(1),$$

and so

$$(22) \quad \| T_{f_k}(r) \leq (k+1)T_f(r) + k \log T_f(r) + k(1 + \varepsilon) \log \log T_f(r) + O(1).$$

Therefore

$$(23) \quad \| \mathbb{C}(0; r; \log g) \leq \sum_{k=1}^{n-1} Y_{t_k}(r) + O(\log^+ T_f(r)) + O(\varepsilon \log r).$$

Thus Theorem 1.1 follows from Theorem 2.5 and (23).

4. NOTES

We define $n+1$ holomorphic vector fields $Z_0, Z_1, \dots, Z_n$ over $\mathbb{P}^n$ as follows. We consider the natural projection

$$\mathbb{P} : \mathbb{C}^{n+1} - \{0\} \longrightarrow \mathbb{P}^n$$

defined by $(\xi_0, \dots, \xi_n) \mapsto [\xi_0, \dots, \xi_n]$, which induces the holomorphic differential

$$\mathbb{P}' : \mathbf{T}(\mathbb{C}^{n+1} - \{0\}) \longrightarrow \mathbf{T}(\mathbb{P}^n).$$

Define

$$Z_i = \mathbb{P}' \left(\frac{\partial}{\partial \xi_i} \right), \quad i = 0, 1, \dots, n.$$

Then $Z_0, \dots, Z_n$ span the holomorphic tangent space at every point of $\mathbb{P}^n$. For example, denoting the local coordinates

$$z_i = \frac{\xi_i}{\xi_0}, \quad i = 1, \dots, n$$

on the domain $U_0 = \{\xi_0 \neq 0\}$ of $\mathbb{P}^n(\mathbb{C})$, we have

$$Z_i = \begin{cases} \frac{1}{\xi_0} \partial_i, & \text{if } 1 \leq i \leq n, \\ -\sum_{j=1}^n \frac{\xi_j}{\xi_0^2} \partial_j, & \text{if } i = 0, \end{cases}$$

where

$$\partial_i = \frac{\partial}{\partial z_i}, \quad i = 1, \dots, n.$$

Now the holomorphic curve

$$f = [\mathfrak{f}_0, \dots, \mathfrak{f}_n] : \mathbb{C} \longrightarrow \mathbb{P}^n$$

satisfies

$$f' = \sum_{i=0}^n \mathfrak{f}'_i Z_i,$$

and so

$$f'_f = \sum_{i=0}^n \mathfrak{f}'_i Z_{if}.$$

Similarly, the associated holomorphic curves

$$f_k = [\mathfrak{f}_0^{k+1}, \mathfrak{f}_0^{k+1} \mathfrak{g}_1^{(k)}, \dots, \mathfrak{f}_0^{k+1} \mathfrak{g}_n^{(k)}] : \mathbb{C} \longrightarrow \mathbb{P}^n$$

satisfy

$$f'_k = (k+1)\mathfrak{f}_0^k \mathfrak{f}'_0 Z_0 + \sum_{i=1}^n \left(\mathfrak{f}_0^{k+1} \mathfrak{g}_i^{(k)} \right)' Z_i$$

with

$$f'_{k,f_k} = (k+1)\mathfrak{f}_0^k \mathfrak{f}'_0 Z_{0f_k} + \sum_{i=1}^n \left(\mathfrak{f}_0^{k+1} \mathfrak{g}_i^{(k)} \right)' Z_{if_k}.$$

Set

$$t_k(Z_{if_k}) = \sum_{j=0}^n a_{kij} Z_{jf}.$$

We make further computations on the domain U_0 , and find

$$f' = \sum_{i=1}^n \mathfrak{g}'_i \partial_i, \quad f'_k = \sum_{i=1}^n \mathfrak{g}_i^{(k+1)} \partial_i,$$

and hence

$$f'_f = \sum_{i=1}^n \mathfrak{g}'_i \partial_{if}, \quad f'_{k,f_k} = \sum_{i=1}^n \mathfrak{g}_i^{(k+1)} \partial_{if_k}.$$

Note that

$$t_k(\partial_{if_k}) = \sum_{j=1}^n (a_{kij} \mathfrak{f}_0^k - a_{kio} \mathfrak{f}_0^{k-1} \mathfrak{f}_j) \partial_{jf},$$

and so

$$t_k(f'_{k,f_k}) = \sum_{j=1}^n \sum_{i=1}^n (a_{kij} \mathfrak{f}_0^k - a_{kio} \mathfrak{f}_0^{k-1} \mathfrak{f}_j) \mathfrak{g}_i^{(k+1)} \partial_{jf}.$$

If we define

$$\mathfrak{g}_{jk} = \begin{cases} \mathfrak{g}'_j, & \text{if } k = 0, \\ \sum_{i=1}^n (a_{kij} \mathfrak{f}_0^k - a_{kio} \mathfrak{f}_0^{k-1} \mathfrak{f}_j) \mathfrak{g}_i^{(k+1)}, & \text{if } 1 \leq k \leq n-1, \end{cases}$$

then the non-degeneracy condition (3) is just equivalent to

$$(24) \quad \det(\mathfrak{g}_{jk}) \neq 0.$$

Set $f_0 = f$. Since every holomorphic vector bundle on $\mathbb{C}$ is trivial, we may choose holomorphic trivialization

$$(25) \quad h_k : f_k^*(\mathbf{T}(\mathbb{P}^n)) \longrightarrow \mathbb{C}^{n+1}$$

and construct the homomorphisms

$$t_k = h_0^{-1} \circ h_k, \quad k = 1, \dots, n-1.$$

Let $e_1, \dots, e_n$ be the standard unit vectors of $\mathbb{C}^n$. If we can choose $e_0 \in \mathbb{C}^n$ such that for each $k = 0, \dots, n-1$, there exists a holomorphic trivialization (25) satisfying

$$h_k(Z_{jf_k}(z)) = (z, e_j), \quad z \in \mathbb{C}, \quad j = 0, 1, \dots, n,$$

we have

$$t_k \left(\sum_{j=0}^n z_j Z_{jf_k}(z) \right) = \sum_{j=0}^n z_j Z_{jf}(z).$$

Thus by using the coordinates on U_0 , we find

$$|t_k(f'_{k,f_k})| = \sum_{\alpha, \beta} \mathfrak{g}_\alpha^{(k+1)} \overline{\mathfrak{g}_\beta^{(k+1)}} h_{\alpha\beta} \circ f,$$

but

$$|f'_k| = \sum_{\alpha, \beta} \mathfrak{g}_\alpha^{(k+1)} \overline{\mathfrak{g}}_\beta^{(k+1)} h_{\alpha\beta} \circ f_k.$$

Hence the key point estimating $Y_{t_k}(r)$ is how to compare $h_{\alpha\beta} \circ f_k$ with $h_{\alpha\beta} \circ f$ sharply.

Griffiths' conjecture would yield the following problem (cf. [10]):

Conjecture 4.1. *Let $f : \mathbb{C} \rightarrow \mathbb{P}^n$ be a holomorphic curve omitting a non-singular divisor without repeated components of degree at least $n + 2$. Then the image of f lies in a proper algebraic subvariety of $\mathbb{P}^n$.*

For example, take a positive integer $d \geq 4$, let P be a non-constant polynomial on $\mathbb{C}$ and consider the holomorphic curve

$$f = [1, e^P, e^{\frac{d}{d-1}P}] : \mathbb{C} \rightarrow \mathbb{P}^2.$$

Green [11] used this holomorphic curve to show that f omits the divisor

$$D = \{\xi_0 \xi_2^{d-1} = \xi_1 (\xi_1^{d-1} - \xi_0^{d-1})\},$$

but is contained in the hypersurface

$$N = \{\xi_1^d = \xi_0 \xi_2^{d-1}\}.$$

It is easy to check that $f_1(\mathbb{C}) \not\subseteq D$. Theorem 1.1 now claims that there is no admissible homomorphism

$$t_1 : f_1^*(\mathbf{T}(\mathbb{P}^2)) \rightarrow f^*(\mathbf{T}(\mathbb{P}^2))$$

satisfying

$$f'_f \wedge t_1(f'_{1, f_1}) \neq 0.$$

Finally, we give an application of jet bundles. Let N be a smooth complex projective variety of dimension n , let k be a positive integer, let $J_k(N)$ denote the holomorphic jet bundle of all k -jets of N so that $J_1(N)$ is the holomorphic tangent bundle $\mathbf{T}(N)$ of N , let $f : \mathbb{C} \rightarrow N$ be a holomorphic curve, and let $j_k(f) : \mathbb{C} \rightarrow J_k(N)$ denote the canonical lifting of f . According to the proof of Theorem 1.1, Theorem 2.5 then easily yields the following result:

Theorem 4.2. *Let N be a smooth complex projective variety of dimension n and let L be a positive line bundle over N . Take*

$$s_j \in \Gamma(N, L) - \{0\}, \quad j = 1, \dots, q,$$

such that the divisor $D = D_1 + \dots + D_q$ has simple normal crossings in N , where $D_j = (s_j = 0)$ is the zero divisor of s_j . Let $f : \mathbb{C} \rightarrow N$ be a non-constant holomorphic curve with $f(\mathbb{C}) \not\subseteq \text{supp}(D)$ such that there exist holomorphic bundle mappings

$$(26) \quad t_k : J_k(N) \rightarrow \mathbf{T}(N), \quad k = 2, \dots, n,$$

satisfying

$$(27) \quad f' \wedge t_2(j_2(f)) \wedge \dots \wedge t_n(j_n(f)) \neq 0.$$

Then for any $\varepsilon > 0$,

$$(28) \quad \begin{aligned} \| \quad qT_f(r, L) + T_f(r, K_N) &\leq \sum_{j=1}^q N_f(r, D_j) + \sum_{k=2}^n \mathbb{C}\langle 0; r; \log |t_k(j_k(f))| \rangle \\ &\quad - N_{\text{Ram}}(r, f) + O(\log^+ T_f(r, L)) + O(\varepsilon \log r). \end{aligned}$$

To estimate the terms $\mathbb{C}\langle 0; r; \log |t_k(j_k(f))| \rangle$ in (28), we need the lemma of logarithmic derivatives due to P. M. Wong [36] (or see [37], [5], [32], [33]):

Theorem 4.3. *Let N be a projective variety and let*

- (i) *D be an effective divisor with simple normal crossings or*
- (ii) *D be the trivial divisor in N (i.e. the support of D is empty or, equivalently, the line bundle associate to D is trivial).*

Let $f : \mathbb{C} \rightarrow N$ be a holomorphic curve and let L be any ample line bundle on N . Then there exists a singular jet metric ρ_k on $J_k(N)(-\log D)$ satisfying

$$(29) \quad \| \quad \mathbb{C}\langle 0; r; \log^+ \rho_k(j_k(f)) \rangle \leq O(\log^+ T_f(r, L)) + O(\varepsilon \log r).$$

The metric ρ_k is constructed as follows: Since there exists a finite number of rational functions $R_1, \dots, R_p$ on N such that the logarithmic jet differentials

$$\left\{ \left(\frac{d^{(j)} R_i}{R_i} \right)^{\frac{m}{j}} \mid 1 \leq i \leq p, 1 \leq j \leq k \right\}$$

span the fibers of the sheaf $\mathcal{J}_k^m(N)(\log D)$ of k -jet differentials of weight m over every point of N (see [37], the proof of Theorem A1), the metric ρ_k is defined by

$$\rho_k(\xi) = \sum_{i=1}^p \sum_{j=1}^k \left| \left(\frac{d^{(j)} R_i}{R_i} \right)^{\frac{m}{j}}(\xi) \right|^2, \quad \xi \in J_k(N)(-\log D).$$

Next we use Theorem 4.3 to the divisor D of Theorem 4.2, and further, assume that the holomorphic bundle mappings (26) decrease the metrics ρ_k , that is,

$$(30) \quad \rho_1(t_k(\xi)) \leq \lambda_k \rho_k(\xi)^{\alpha_k}, \quad \xi \in J_k(N), \quad k = 2, \dots, n,$$

for some positive constants λ_k and α_k . From the choice of R_i and the express of $\text{Ric}(\Psi)$ (see the proof of Lemma 6.21 in [19]), it is easy to find a constant c' such that

$$(31) \quad h_\Psi \leq c' \rho_1,$$

and hence

$$|t_k(j_k(f))| \leq c' \rho_1(t_k(j_k(f))), \quad k = 2, \dots, n.$$

Therefore,

$$|t_k(j_k(f))| \leq c' \lambda_k \rho_k(j_k(f))^{\alpha_k}, \quad k = 2, \dots, n,$$

since the bundle mappings t_k decrease the metrics ρ_k . By using Theorem 4.3, we obtain

$$(32) \quad \sum_{k=2}^n \mathbb{C}\langle 0; r; \log |t_k(j_k(f))| \rangle \leq O(\log^+ T_f(r, L)) + O(\varepsilon \log r),$$

and hence (28) yields

$$(33) \quad \begin{aligned} \| \quad qT_f(r, L) + T_f(r, K_N) &\leq \sum_{j=1}^q N_f(r, D_j) - N_{\text{Ram}}(r, f) \\ &+ O(\log^+ T_f(r, L)) + O(\varepsilon \log r). \end{aligned}$$

ACKNOWLEDGEMENT

The authors want to thank the anonymous referee's many significant helpful remarks and suggestions.

REFERENCES

- [1] Biancofiore, A., A hypersurface defect relation for a class of meromorphic maps, *Trans. Amer. Math. Soc.* 270 (1982), 47-60. MR642329 (83h:32005)
- [2] Bloch, A., Sur les systèmes de fonctions holomorphes à variétés linéaires lacunaires, *Ann. Ecole. Norm. Sup.* 43 (1926), 309-362. MR1509274
- [3] Carlson, J. and Griffiths, P., Defect relation for equidimensional holomorphic mappings between algebraic varieties, *Ann. of Math.* 95 (1972), 557-584. MR0311935 (47:497)
- [4] Cartan, H., Sur les systèmes de fonctions holomorphes à variétés linéaires lacunaires, *Ann. Sci. Ecole Norm. Sup.* 45 (1928), 255-346. MR1509288
- [5] Chandler, K. and Wong, P. M., Finsler geometry of holomorphic jet bundles, in "A Sampler in Riemann/Finsler Geometry", MSRI Publ. 50, Cambridge University Press (2004), 107-196. MR2132659 (2006i:32022)
- [6] Dufresnoy, H., Théorie nouvelle des familles complexes normales. Applications à l'étude des fonctions algébroides, *Ann. Sci. Ecole Norm. Sup.* 61 (1944), 1-44. MR0014469 (7:289f)
- [7] Fujimoto, H., Extensions of the big Picard theorem, *Tohoku Math. J.* 24 (1972), 415-422. MR0333253 (48:11578)
- [8] Fujimoto, H., Families of holomorphic maps into the projective space omitting some hyperplanes, *J. Math. Soc. Japan* 25 (1973), 235-249. MR0328131 (48:6473)
- [9] Green, M., Holomorphic maps into complex projective space omitting hyperplanes, *Trans. Amer. Math. Soc.* 169 (1972), 89-103. MR0308433 (46:7547)
- [10] Green, M., Some Picard theorems for holomorphic mappings to algebraic varieties, *Amer. J. Math.* 97 (1975), 43-75. MR0367302 (51:3544)
- [11] Green, M., Some examples and counter examples in value distribution theory of several complex variables, *Composito Math.* 39 (1975), 317-322. MR0374498 (51:10698)
- [12] Green, M. and Griffiths, P., Two applications of algebraic geometry to entire holomorphic mappings, *The Chern Symposium 1979 (Proc. Internat. Sympos., Berkeley, Calif., 1979)*, Springer-Verlag, New York, 1980, pp. 41-74. MR609557 (82h:32026)
- [13] Griffiths, P., Holomorphic mappings: Survey of some results and discussion of open problems, *Bull. Amer. Math. Soc.* 78 (1972), 374-382. MR0294718 (45:3786)
- [14] Griffiths, P. and King, J., Nevanlinna theory and holomorphic mappings between algebraic varieties, *Acta Math.* 130 (1973), 145-220. MR0427690 (55:721)
- [15] Hu, P. C., Holomorphic mapping into algebraic varieties of general type, *Nagoya Math. J.* 120 (1990), 155-170. MR1086576 (92b:32033)
- [16] Hu, P. C., Holomorphic mappings between spaces of different dimensions I, *Math. Z.* 214 (1993), 567-577; II, *Math. Z.* 215 (1994), 187-193. MR1248113 (94k:32043); MR1259455 (95a:32046)
- [17] Hu, P. C., P. Li and Yang, C. C., Unicity of meromorphic mappings, *Kluwer*, 2003. MR1988768 (2004m:32031)
- [18] Hu, P. C. and Yang, C. C., Meromorphic functions over non-Archimedean fields, *Mathematics and Its Applications* 522, Kluwer Academic Publishers, 2000. MR1794326 (2002a:11085)
- [19] Hu, P. C. and Yang, C. C., Value distribution theory related to number theory, *Birkhäuser*, 2006. MR2245631 (2008c:32021)
- [20] Kobayashi, S., Hyperbolic complex spaces, *GMW* 318, Springer, 1998. MR1635983 (99m:32026)

- [21] Lang, S., Introduction to complex hyperbolic spaces, Springer-Verlag, 1987. MR886677 (88f:32065)
- [22] Nadel, A. M., Hyperbolic surfaces in $\mathbb{P}^3$, Duke Math. J. 58 (1989), 749-771. MR1016444 (91a:32036)
- [23] Shiffman, B., Holomorphic curves in algebraic manifolds, Bull. Amer. Math. Soc. 83 (1977), 553-568. MR0440075 (55:12956)
- [24] Siu, Y. T., Defect relations for holomorphic maps between spaces of different dimensions, Duke Math. J. 55 (1) (1987), 213-251. MR883671 (89a:32030)
- [25] Siu, Y. T., Nonequidimensional value distribution theory and meromorphic connections, Duke Math. J. 61 (2) (1990), 341-367. MR1074301 (92f:32044)
- [26] Siu, Y. T., Nonequidimensional value distribution theory, Lecture Notes in Math. 1351, 285-311, Springer-Verlag, 1988. MR982094 (90j:32028)
- [27] Siu, Y. T. and Yeung, S. K., Defects for ample divisors of Abelian varieties, Schwarz lemma, and hyperbolic hypersurfaces of low degrees, Amer. J. Math. 119 (1997), 1139-1172. MR1473072 (98h:32044)
- [28] Siu, Y. T. and Yeung, S. K., Addendum to "Defects for ample divisors of Abelian varieties, Schwarz lemma, and hyperbolic hypersurfaces of low degrees", Amer. J. Math. 125 (2003), 441-448. MR1980629 (2004e:32028)
- [29] Stoll, W., Value distribution on parabolic spaces, Lecture Notes in Math. 600 (1977), Springer-Verlag. MR0450626 (56:8919)
- [30] Stoll, W., The Ahlfors-Weyl theory of meromorphic maps on parabolic manifolds, Lecture Notes in Math. 981 (1983), 101-219, Springer-Verlag. MR699135 (85c:32045)
- [31] Vojta, P., Diophantine approximation and value distribution theory, Lecture Notes in Math. 1239, Springer, 1987. MR883451 (91k:11049)
- [32] Vojta, P., On the ABC conjecture and Diophantine approximation by rational points, Amer. J. Math. 122 (2000), 843-872. MR1771576 (2001i:11094)
- [33] Vojta, P., Correction to: "On the ABC conjecture and Diophantine approximation by rational points" [Amer. J. Math. 122 (2000), no. 4, 843-872; MR1771576 (2001i:11094)]. Amer. J. Math. 123 (2001), no. 2, 383-384. MR1828227 (2002d:11095)
- [34] Wang, J. T. Y., The truncated second main theorem of function fields, J. Number Theory 58 (1) (1996), 139-157. MR1387732 (97c:11074)
- [35] Wong, P. M. and Wong, Philip P. W., The second main theorem on generalized parabolic manifolds, preprint.
- [36] Wong, P. M., Nevanlinna theory for holomorphic curves in projective varieties, preprint.
- [37] Wong, P. M. and Stoll, W., On holomorphic jet bundles, math. AG/0003226, preprint.
- [38] Wu, H., The equidistribution theory of holomorphic curves, Princeton University Press, Princeton, New Jersey, 1970. MR0273070 (42:7951)
- [39] Ye, Z., On Nevanlinna's second main theorem in projective space, Invent. Math. 122 (1995), 475-507. MR1359601 (96j:32030)

DEPARTMENT OF MATHEMATICS, SHANDONG UNIVERSITY, JINAN 250100, SHANDONG, PEOPLE'S
REPUBLIC OF CHINA

E-mail address: pchu@sdu.edu.cn

DEPARTMENT OF MATHEMATICS, THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY,
HONG KONG, PEOPLE'S REPUBLIC OF CHINA

E-mail address: chungchun.yang@gmail.com